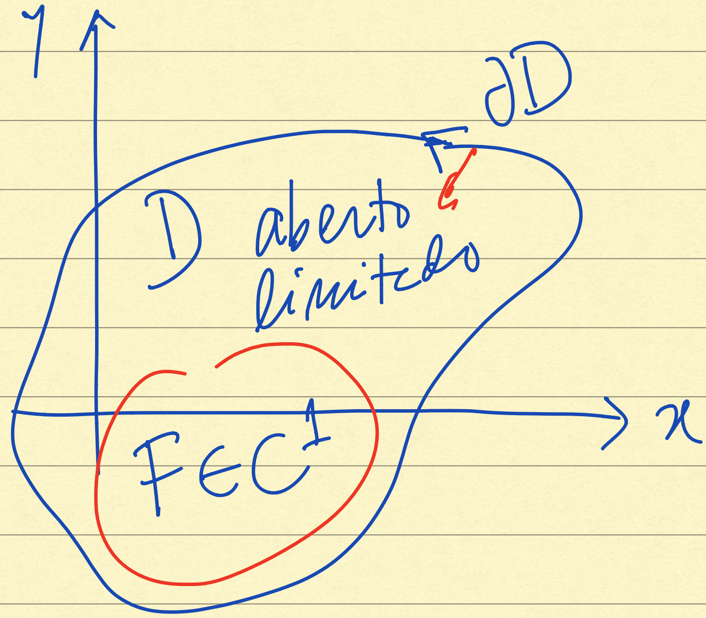
Teorema de Green: ($\mathbb{R}^2$)



$$\boxed{F: \mathbb{R}^2 \to \mathbb{R}^2, C^1}$$

$$F = (P, Q)$$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \oint_{\partial D} P\,dx + Q\,dy$$

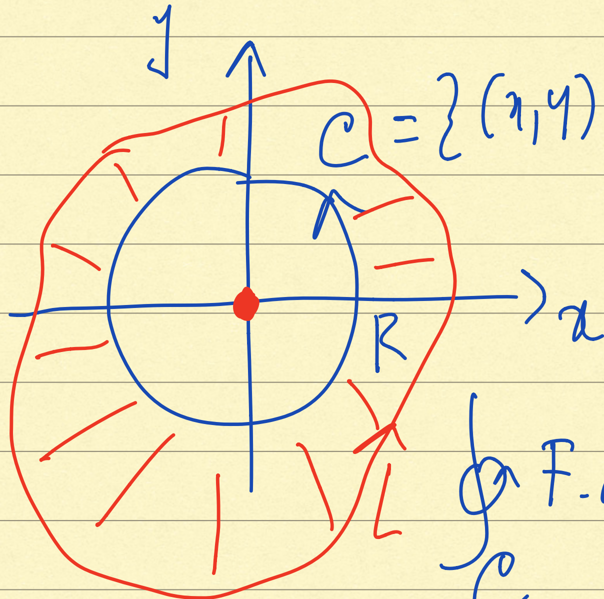
$$\oint_{\partial D} F \cdot d\gamma$$

$\boxed{\text{Trabalho}}$

Exemplo: $F(x,y) = \left(\overset{P}{-\frac{y}{x^2+y^2}}, \overset{Q}{\frac{x}{x^2+y^2}}\right)$

Fechado!

1) $(x,y) \neq (0,0)$



$C = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 = R^2\}$

contas

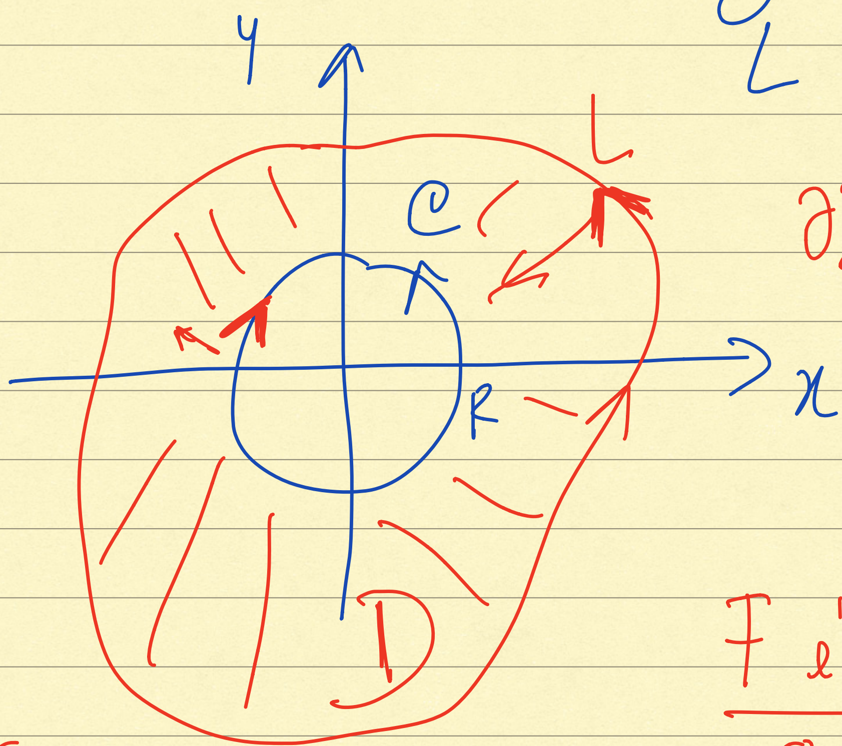
$$\oint_C F \cdot d\gamma = 2\pi, \ \forall R>0$$

$D \equiv$ conjunto limitado por L e C.

$F \in C^1$ em D.

$\partial D = L \cup C$

Sabendo $\oint_C F \cdot dg = 2\pi$, queremos encontrar o valor de $\oint_L F \cdot dg$.



$\partial D = L \cup C$

$\oint_C F \cdot dg = -2\pi$

$\underline{F \text{ é fechado:}}$

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

$$\boxed{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0}$$

Green $\Rightarrow$

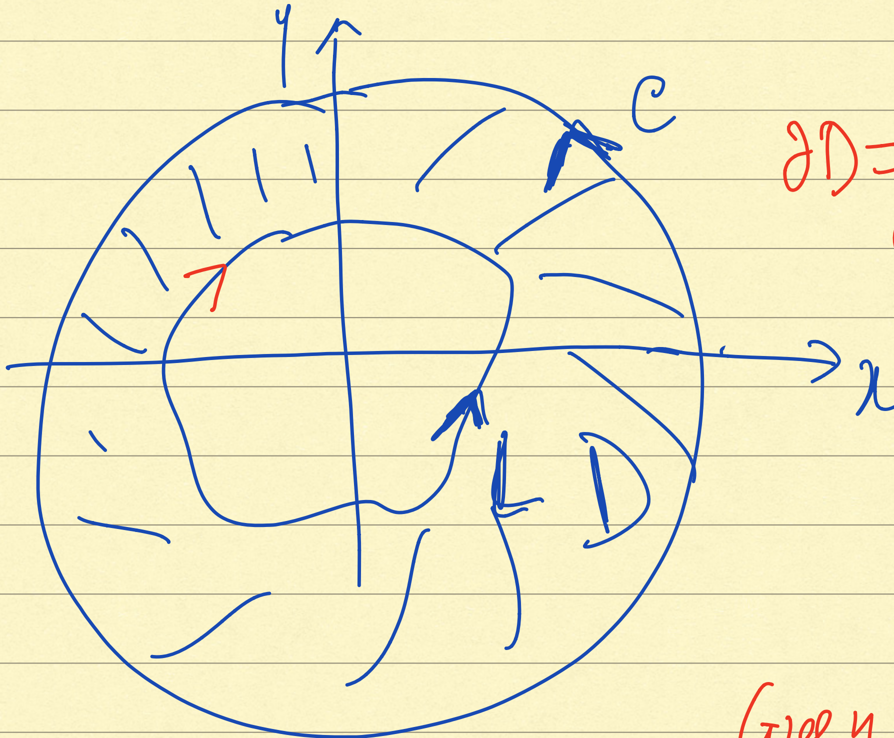
$$0 = \iint_D \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{=0} dx\,dy \overset{\text{T. Green}}{=}$$

$$= \oint_L P\,dx + Q\,dy + \underbrace{\oint_C P\,dx + Q\,dy}_{-2\pi \text{ (contro)}}$$

$$0 = \oint_L P\,dx + Q\,dy - 2\pi$$

$$\therefore \boxed{\oint_L P\,dx + Q\,dy = 2\pi}$$

Or



$\partial D = C \cup L$

$\circlearrowleft \; \circlearrowright$

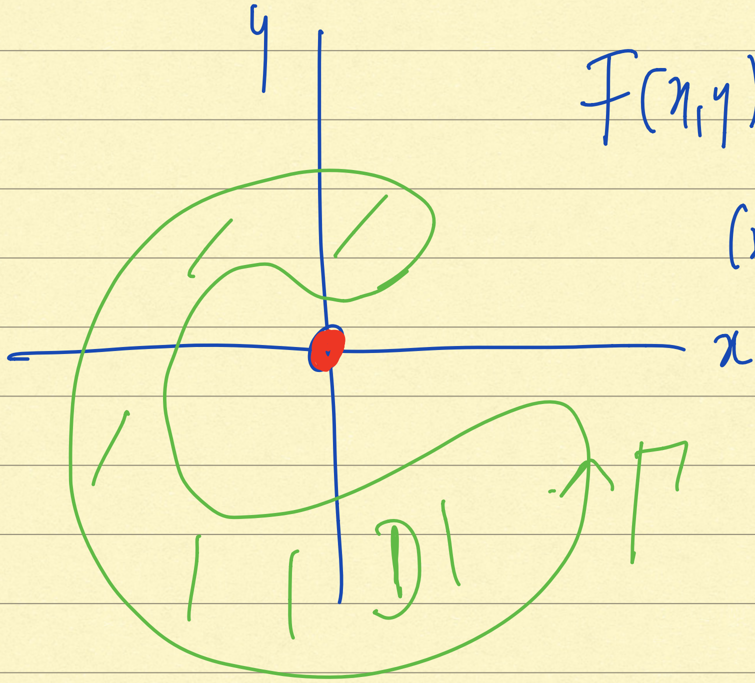
Green

$$0 = \iint_D \Big(\underbrace{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}_{=0}\Big)\,dx\,dy = \oint_{\partial D} P\,dx + Q\,dy$$

$$= \underbrace{\oint_C P\,dx + Q\,dy}_{2\pi} + \oint_L P\,dx + Q\,dy$$

$$= 2\pi - \oint_L P\,dx + Q\,dy$$

2)



$F(x,y) = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$

$(x,y) \neq \boxed{(0,0)}$

"gama"

$D \equiv$ conjunto limitado por Γ.

$\partial D \equiv \Gamma$.

Green:

$$0 = \iint_D \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{=0} dx\,dy \overset{\text{Green}}{=} \oint_{\boxed{\Gamma}} P\,dx + Q\,dy$$

1)

$\oint_L F = 2\pi$ (1 volta)

2) $\oint_L F = 0$

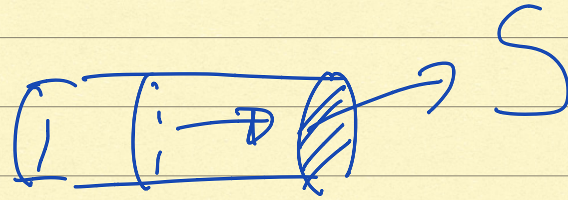
1 ponto P $\equiv$ trajetória c/ velocidade zero.

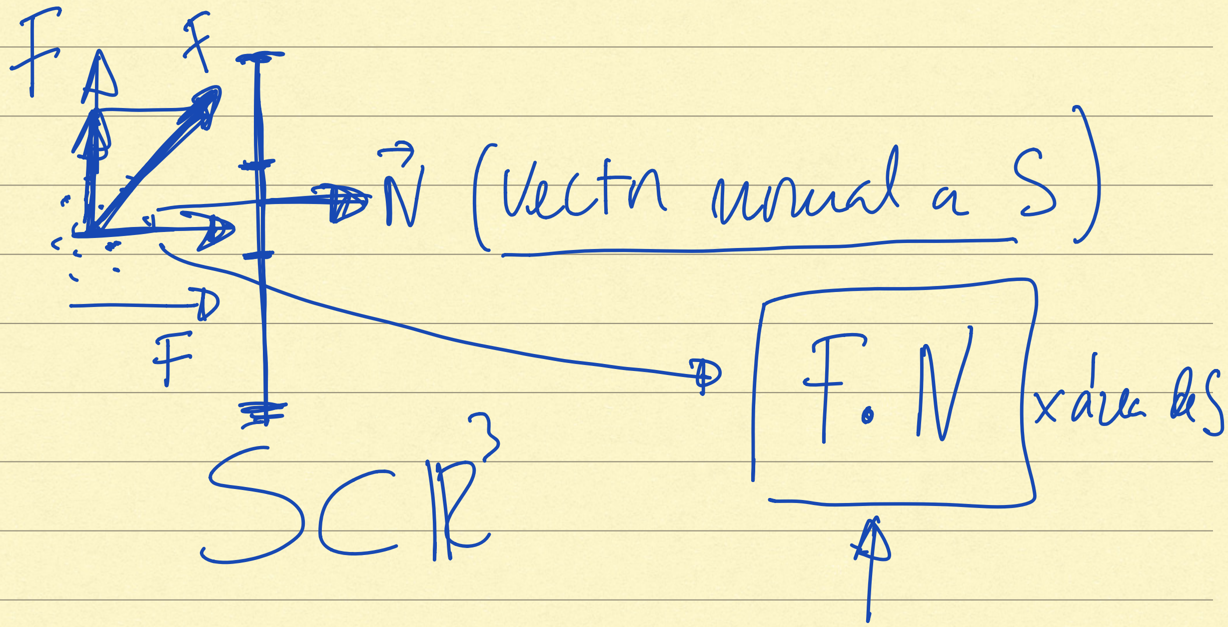
$$g(t) = P$$

$$g'(t) = 0$$

$$\int F(g(t)) \cdot \underbrace{g'(t)}_{=0} dt = 0 \; !$$

Fluxo





Fluxo de partículas através de S provocado por F é proporcional à componente de F na direcção normal a S (F.N) e proporcional à área de S.